

PRESENTATION THEOREMS FOR CODED CHARACTER SETS

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Abstract.

The notion of 'presentation', as used in combinatorial group theory, is applied to coded character sets(CCSs) – sets which facilitate the interchange of messages in a digital computer network(DCN) . By grouping each element of the set into two portions and using the idea of group presentation(whereby a group is specified by its set of generators and its set of relators) , the presentation of a CCS is described. This is illustrated using the Extended Binary Coded Decimal Interchange Code(EBCDIC) which is one of the most popular CCSs in DCNs.

Key words. Group presentation, coded character set, digital computer network

1. Introduction.

Most of the data which are presented to a digital computer system(DCS) are in the form of records and these records are usually entered as alphanumeric characters. For every number which is manipulated by a DCS, ten alphabetic characters are processed [13]. Mathematically, a coded character set(CCS) arises as a result of a mapping between the set of binary digits(bits) and the set of characters ; the emphasis in this work is on the set of bits(codes) . Thus, the set of bits of a CCS, and in effect a CCS, is structurally a set of sequences. The ordering in the set is according to the collating sequence(i.e the natural sequence of appearance in the set). Coded character sets(CCSs) are important in a DCS because they provide a means of representing alphanumeric characters(i.e numerals, letters, punctuation characters and control codes) as fixed sequences of zeros and ones. The sets are normally utilized in the sixth layer(i.e presentation layer) of the seven open system interconnection(OSI) layers of computer network [34]. Two popular examples of such sets are the 7 – *bit* American Standard Code for Information Interchange(ASCII) consisting of 128 characters and the Extended Binary Coded Decimal Interchange Code(EBCDIC) consisting of 256 characters. Although a CCS is a code(i.e it is a code mapped unto a set of characters), it is not an error detection or correction(EDC) code and also, it is not always a group code . A well-known result is that a code is linear iff it is a group [4] . A lot of the literature has been devoted to studies of the mathematical properties of linear and nonlinear codes and of the construction of EDC codes e.g see [1, 10, 11, 19]. In particular, some properties of group codes are discussed in [21]. Also, traditional studies on the properties of CCSs have focused on the description of the characteristics of the codes in terms of shiftedness, BCD for numerics, BCD for alphabets, numerics in numeric sequence, signed numerics and the matching of collating sequence with the bit sequence [25]. For instance, the EBCDIC is not shifted and its alphabets are not in contiguous sequence. However, it possesses all the other properties mentioned above . In some of his earlier papers [e.g 29, 31], the author applied the idea of equivalence relation to ordinary differential equations of the form $x' = g(x) = \sum_{i=1}^n a_i x^{n-i}, a_i \in \mathfrak{R}$ and constructed codes using the quaternary system $\{Attractor(A), Repellor(R), PositiveShunt(P), NegativeShunt(N)\}$ where A, R, P and N are the possible phase portraits for (a linear) equation on the line. The blocklength of a code constructed this way is equal to the degree n of the polynomial. In [6], a geometric technique for constructing codes

using the *black/white* lift of a cap was presented. This is accomplished by partitioning a set of points which have no three of its points collinear. In the present paper however, a new approach to the study of CCSs (and binary uniform digital codes in general) is discussed and pertinent theorems on the approach presented. The approach, called 'code presentation', uses the concept of a partition and an imitation of the idea behind 'group presentation'. This enables a CCS to be described in a simple form in terms of a 'zoned set' and a 'decinumer set' just as a group is described in terms of a 'set of generators' and a 'set of relators'[7, 12, 14, 18, 22, 24, 26, 27, 33]. In particular, it is shown in [27] that a generalized free product of two finitely presented groups acting on trees(a non-primitive computer data structure) is finitely presented iff the amalgamated subgroup is finitely generated. The general applicability of the technique of code presentation to codes had been discussed in [30]. As is well-known, the idea behind (group) presentation, in itself, has been applied to a number of areas in mathematics and computer science including geometry, topology, C^* -algebra, knot theory, automorphic functions [26], computable algebra [15] and other areas. Presentation in its ordinary meaning refers to the depicting or writing of an algebraic structure in a simple form. In particular, in the Theory of Computation, the presentation of a function refers to a definition which gives an effective method for computing the function [5]. In Formal Language Theory, the concept of 'generation'(or derivation) is associated with the phenomenon in which a language may be generated by a phrase structure grammar [16]. Although the term 'Source Code Presentation' exists in Software Engineering, it is used in a different setting to describe the readability of the source code of a program [35]. The beauty in group presentation, apart from its simplicity, is that it assists in deriving information about an algebraic structure from its presentation.

2. Group Presentation.

Code presentation may be indirectly viewed as another area of application of group theory. Generally, group presentation per se is unsuitable for presenting CCSs since not all binary uniform digital codes $(C, +)$ are groups where C is a typical code and $+$ the binary operation defined on it. That is, the four basic axioms of a group(namely closure, existence of an identity, existence of an inverse and associativity) are not always satisfied in a code

. For instance, the simple code $\{00000, 01100, 00110, 11000, 11001, 11011\}$ of order six and blocklength five is not a group because it is not closed. The methodology of code presentation used in this paper is based on the premise that a typical modern day digital computer has an architecture in which the bit patterns of a memory location in the main memory are addressed in bytes i.e in multiples of bits. The following are well-known results on group presentation [8, 26] :

Theorem 2.1.

- (i) Every group has at least one set of generators, namely itself .
- (ii) Every group necessarily has a presentation. In particular, every finite group has a finite presentation.
- (iii) A subgroup of a finitely presented group need not be finitely presented .
- (iv) A group can have many presentations .
- (v) Given an arbitrary set of symbols and an arbitrarily prescribed set of words in these symbols, there exists a unique group, up to isomorphism, having the symbols as generators and the set of prescribed words as defining relators. (This result always allows new groups to be constructed).

Two examples of group presentation satisfying the above are:

- (i) Symmetric group of degree n (S_n) [22]

$$S_n \approx \{\sigma_1 \sigma_2 \dots \sigma_{n-1}; I_n \cup B_n\}$$

where I_n is the set of relations $\sigma_i^2 = 1$ for $i \in \{1, 2, \dots, n-1\}$ and B_n is the set of relation $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$ if $i \in \{1, 2, \dots, n-1\}$ and $\sigma_i \sigma_j = \sigma_j \sigma_i$ if $i, j \in \{1, 2, \dots, n-1\}$ and $|i - j| \geq 2$. Under the above isomorphism, σ_i is taken to be the transposition $(i, i+1)$. In particular, the presentation of the symmetric group on three letters (S_3) is [3]

$$S_3 \approx \{a, b; a^2, b^3, a^{-1}bab^{-2}\}$$

- (ii) The Mathieu group M_{11} [12]

(a)

$$\{a, b, c, d, e; aa, bb, cc, dd, ee, bdbd, bebe, (ab)^3, (de)^3, (bc)^5, acece, a(cd)^3\}$$

(b)

$$\{a, b, c, d; aa, bb, cc, dd, (ab)^3, (bc)^3, (cd)^3, (abdbd)^2, (cbdbd)^3\}$$

(c)

$$\{a, b, c, d; aa, bb, cc, dd, acac, adad, (ab)^3, (bc)^3, (cd)^3, (bd)^5, (abd)^5, (bcdcbdc)^3\}$$

3. Code Presentation Theorems.

Let $w = w_1w_2$ be a juxtaposed word of a uniform digital code C of order k and blocklength n such that $w_1 = a_1a_2\dots a_s$ and $w_2 = a_{s+1}a_{s+2}\dots a_n$. Then w_1 and w_2 are respectively called the zoned portion and numeric portion of w . If n is even we let $s = n/2$ and if n is odd, we let $s = (n+1)/2$ or $s = (n-1)/2$. The Type I definition of zoned portion for odd n is depicted by the case when $s = (n+1)/2$ while the case in which $s = (n-1)/2$ describes the Type II definition of zoned portion. A constant zoned portion refers to a zoned portion which is the same for two or more words. Let the ordering on the code be according to the collating sequence. Then the ordered set of all the constant zoned portions of C is called the zoned code (or zoned set). The numer code (or numer set) of C is the ordered set of all the numeric portions of the code. We now suppose E is a subset of C . Then E is said to be an equizone of C if all words in E have a single constant zoned portion. The degree of C refers to the number of equizones in it. A decinumer of E refers to the decimal value of a numeric portion of E while the ordered set of all the decinumeres of an equizone is called a decinumer set [30].

Theorem 3.1.

Every coded character set C can be well-ordered.

Proof.

A CCS is a set. By the well-ordering principle [20], every set can be well-ordered. For $c_1, c_2 \in C$, define $c_1 \leq c_2$ iff the decimal value of c_1 is less than or equal to the decimal value of c_2 based on the collating sequence of C . It is easily seen that C is a chain and every subset of it contains a first element. Therefore, a CCS can be well-ordered.

Definition 3.2(Fundamental Definition of Code Presentation).

Let a uniform digital code C has a degree d and suppose E_i is a typical equizone of decinumer set Q_i where $i < d$. If z_i is the constant zoned portion

of E_i and x_{ig} the bit pattern of $g \in Q_i$, then the code presentation of C is given by:

$$C\{P\} = \bigcup_{i=1}^d \{z_i x_{ig} \forall g \in Q_i\} \quad (3.2)$$

Definition 3.3.

Let $w_1 = a_{11}a_{12}\dots a_{1n}$ and $w_2 = a_{21}a_{22}\dots a_{2n}$ be two words of a coded character set and $'*'$ a binary operation. Then $w_1 * w_2$ is defined as

$$w_1 * w_2 = (a_{11} * a_{21})(a_{12} * a_{22})(a_{1n} * a_{2n}) \quad (3.3a)$$

. In particular, the word difference(-) between w_1 and w_2 , denoted by $w_1 - w_2$, is given by

$$w_1 - w_2 = (a_{11} - a_{21})(a_{12} - a_{22})(a_{1n} - a_{2n}) \quad (3.3b)$$

where

$$a_{ij} - a_{kj} = \begin{cases} 0 & \text{if } a_{ij} = a_{kj} \\ 1 & \text{otherwise} \end{cases}$$

Remark 3.4.

Addition in computer arithmetic is normally defined by: $0 + 0 = 0$; $0 + 1 = 1$; $1 + 0 = 1$ and $1 + 1 = 0$ while multiplication is normally defined by : $0.0 = 0$; $0.1 = 0$; $1.0 = 0$ and $1.1 = 1[9]$

Proposition 3.5.

$$w_1 - w_2 = w_1 + w_2$$

Proof.

Obvious; it follows from the fact that $a_{ij} - a_{kj} = a_{kj} - a_{ij} = a_{ij} + a_{kj}$

Theorem 3.6.

Let C be a uniform digital code and T the zoned code of C. Given $a, b \in C$ and $z_a, z_b \in T$, let $a \sim b$ iff $z_a - z_b = 0$, where z_a, z_b are respectively the zoned portions of a and b and $'-'$ is the word difference. Then \sim defines an equivalence relation

Proof.

This follows from the fact that \sim is reflexive, symmetric and transitive.

The distinct equivalence classes of \sim are the equizones. By virtue of the Fundamental Theorem of Equivalence Relations, it follows that the set of all these equivalence classes gives a partition of the code C .

Definition 3.7.

Let $C = \{X_1, X_2, \dots, X_e\}$ be an ordered code of order e where $X_i = x_{i1}x_{i2}\dots x_{in}$, $x_{ij} \in \{0, 1\}$, $j = 1, 2, \dots, n$ is a word of blocklength n . Then the inverse of C , denoted by C^{-1} , is an ordered code given by $C^{-1} = \{X_n, X_{n-1}, \dots, X_1\}$. The inverse of X_i , denoted by X_i^{-1} , is given by $X_i^{-1} = x_{in}^{-1}x_{in-1}^{-1}\dots x_{i1}^{-1}$ where

$$x_{ij}^{-1} = \begin{cases} 0 & \text{if } x_{ij} = 1 \\ 1 & \text{otherwise} \end{cases}$$

Theorem 3.8.

- (i) Every coded character set (CCS) necessarily has a presentation.
- (ii) The order of the zoned set of a coded character set is finite.
- (iii) Every subset of a coded character set has a finite presentation.
- (iv) The presentation of a coded character set is not necessarily unique.
- (v) A coded character set can have at most two presentations.

Proof.

(i) Since every CCS can be written in terms of a zoned portion and a numeric portion, then the result follows. (ii) A CCS is finite. Therefore it has a finite number of zoned portions. (iii) This follows from the fact that every subset of a CCS is finite. (iv) When a CCS has an odd blocklength, then its zoned portion, by definition, can be described in two ways. It then follows that the CCS has two distinct presentations. (v) The two distinct presentations in the proof of (iv) are the only possible presentations. They are the maximum possible presentations.

4. Example.

The EBCDIC is one of the two most popular CCSs in digital computer systems [2]. It has 16 equizones namely $E_0, E_1, \dots, E_9, E_A, E_B, \dots, E_F$. The zoned set of the code is the 4-bit hexad set $H_{16}^4 = \{0000, 0001, 0010, 0011, \dots, 1110, 1111\}$. Table 1 gives the decinumer set of each equizone of the EBCDIC code where $'-'$ is the set difference. In the table, l_i is the decinumer set

which corresponds to equizone E_i while T_4 represents the order-preserving decinumer set of H_{16}^4 i.e $T_4 = \{0, 1, 2, \dots, 14, 15\}$. Both equizones E_1 and E_2 have the highest number of characters (i.e 13 characters each) while equizone E_B with no character, has the least number of characters.

5. Discussion and Conclusion.

This paper has related the idea behind group presentation to coded character sets (CCSs) in digital computer architecture . Theorems, which are analogies of some of the well-known results in combinatorial group theory, are then presented. Just as group presentation, code presentation enables CCSs to be written in a simple form and also assists in deriving information about their structure. The results in the paper arise from the fact that group presentation theory (or combinatorial group theory), is generally unsuitable for presenting CCSs since not all CCSs are groups. Apart from this, the bits of CCSs have some particular pattern of representation in the classical computer hardware [23, 25].

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TABLE 1: THE DECINUMER SET OF THE EBCDIC

| i | l_i |
|---|----------------------------|
| 0 | $T_4 - \{1, 2, 3, 8, 12\}$ |
| 1 | $T_4 - \{0, 1, 2\}$ |
| 2 | $T_4 - \{3, 8, 9\}$ |
| 3 | $T_4 - \{0, 1, 2, 3\}$ |
| 4 | $T_4 - l_8$ |
| 5 | l_4 |
| 6 | $l_4 \cup \{1\}$ |
| 7 | $l_4 - \{0\}$ |
| 8 | $\{1, 2, \dots, 9\}$ |
| 9 | l_8 |
| A | $l_8 - \{1\}$ |
| B | ϕ |
| C | l_8 |
| D | l_8 |
| E | l_A |
| F | $l_8 \cup \{0, 15\}$ |

LIST OF SYMBOLS

| SYMBOL | MEANING |
|-------------|--|
| \in | is an element of |
| \forall | for all |
| \leq | less than or equal to |
| \cup | union of some sets |
| H_n^{16} | n-bit hexad set |
| $\{\}$ | set of elements |
| $A - B$ | the difference of two sets A and B(set difference) |
| $w_1 - w_2$ | word difference |
| T_4 | order-preserving equidext of H_4^{16} |
| \emptyset | empty set |